

F(750), We Miss You as Bound State of 6 Top and 6 Anti top

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Abstract

We collect and estimate support for our long speculated “multiple point principle” [11, 12, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 37, 26, 27, 28, 29, 30, 31] saying that there should be several vacua all having (compared to the scales of high energy physics) very low energy densities. In pure Standard Model we suggest there being three by “multiple point principle” low energy density vacua, “present”, “condensate” and “high field” vacuum. We fit the mass of the in our picture since long speculated bound state [69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 8] of six top and six anti top quarks in three quite *independent ways* and get remarkably within our crude accuracy the *same* mass in all three fits! The new point of the present article is to estimate the bound state mass in what we could call a bag model estimation. The two other fits, which we review, obtain the mass of the bound state by fitting to the multiple point principle prediction of degenerate vacua.

Our remarkable agreement of our three mass-fits can be interpreted to mean, that we have calculated at the end the energy densities of the two extra speculated vacua and found that they are indeed very small!. Unfortunately the recently much discussed statistical fluctuation peak $F(750)$ [2, 3, 4, 5] has now been revealed to be just a fluctuation, very accidentally matches our fitted mass of the bound state remarkably well with the mass of this fluctuation 750 GeV.

1 Introduction

We have long worked on the speculation, that six top and six anti top quarks due to mainly the rather large value of the top-yukawa coupling g_t and thus to Higgs boson exchange gets bound so strongly to each other, that a bound state with a mass appreciably lower than the sum of the masses of 12 top-quarks is formed[9, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 8, 6]. In the present article we shall put forward an attempt to estimate the mass of this bound state by setting up a kind of bag-model ansatz for the bound state system. The “bag” here denotes a region in space, where the Higgs field is equal to zero, so that the mass of the quarks, e.g. the top quark, is also zero there. Thus such a bag filled with top and anti top quarks can make up a bound state, that is identified as our long speculated one. We consider it of great importance to estimate the mass of this bound state, not only because we hope, that LHC or some further accelerator might find it some day, but also because, if it is as expected strongly bound, it is expected to function as approximately a new elementary particle giving rise to loop diagrams, that can give various corrections. Most important for the trustability of our long speculated picture being based on what we call the “multiple point principle” saying, that there are several

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vacua with very small energy density ¹ is, that calculation of the energy density of candidates for such alternative vacua is sensitive to the mass of our bound state. In our speculation we have in the pure Standard Model - but we have many applications also to extensions of the Standard Model with more speculated vacua - just three vacua, which we may call “present”, “condensate” and “high field”. The “present” stands for the usual vacuum, in which we so to speak live, while “condensate” denotes a state in which space is filled up with a smooth density of the bound states; they may possibly form a bose-condensate, but at least they should be present with some density and interact with each other. In a work with C. D. Froggatt we approximated the configuration of the bound state distribution in the state making up the “condensate” vacuum by the configuration of carbon atoms in a diamond chrystal. Whether the interaction between the bound states present in such a “condensate”-vacuum can just compensate the Eintein-energy(i.e. the mass) of the bound states, is presumably rather much dependent on this mass. Thus by a rather round about way Froggatt and I obtained a mass prediction 285 GeV for the bound state [39]. More directly but basically by the same method I estimate[9] a mass of $4m_t = 690\text{GeV}$ from this requirement that the “condensate” vacuum be degenerate with the ‘present” one. Also with Das and Laperashvili [8] we estimated a rather small correction to the Higgs mass, that should correspond to the second minimum in the Higgs field effective potential - what we called “high field” vacuum - should just touch zero, which is equivalent to the “high field” vacuum having zero(small) energy density. Without any correction from the bound state the Higgs mass, that just makes this boarderline stability, is[63] 129.4 GeV, but we can assuming a bound state mass in the 800 GeV range[8] obtain a correction making the experimental Higgs mass 125 GeV compatible with the degeneracy of the “high field” and the “present” vacua.

The result of the present article from calculating the bag-model mass estimate is, that this estimate comes after all the corrections to be almost unexpectedly close to just the value that is needed for arranging the two alternaitve vacua, the “condensate” and the “high field” one to be degenerate with the “present” vacuum. This would mean a calculational confirmation of our long speculated “Multiple Point Principle”. So we could claim, that if the calculations including the present mass estimate and the previous works concerning the degeneracies of the two alternative vacua got confirmed, to have calculated, that this “Multiple Point Principle” were simply true (by calculation) for the three vacua proposed to be possibilities in a picture of pure Standard Model. If as we claim crudely below the three vacua are indeed degenerate, it would not only imply that we established our - one could say new law of nature - the “Multiple Point Principle”, but also that we would need the Standard Model to work with sufficient accuracy almost all the way to the Planck scale. At least possible disturbances of the pure Standard Model by for instance see-saw neutrinoes or super symmetry or whatever should be so small, when counted in the Higgs effective potential, that it would only change our calculations negligibly.

We must admit that it must be just a mysterious accident, that the average mass of our three remarkably coinciding masses happen to be very close to that mass, which were seen as a digamma $F(750)$ fluctuation[2, 3, 4, 5, 1] - only revealed to be a fluctuation quite recently[7] -.

It has all the time been so, that we have presented our bound state story together with our “Multiple Point Principle” [11, 12, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 37, 26, 27, 28, 29, 30, 31].

Historically we - Don Bennett and I at first - invented this priciples[11, 13, 12, 14] in order to justify a model, in which we fitted fine structure constants by means multiple points in phase

¹Strictly speaking the multiple point principle just tells that there are several vacua with the *same* energy density. But if you instead say that there are several vacua with *very small* energy density, you formally make the mystery of the smallness of the cosmological constant become formally a part of the assumption of our multiple point principle. For this point we thank L. Susskind.

diagrams for lattice gauge theories. This were in a model with each family of fermions having its own family of gauge particles (Anti GUT). [35, 36, 32, 33, 34]

We shall understand the Multiple Point Principle as a principle, that delivers restrictions between the coupling constants and mass parameters (the bare couplings or renormalized ones does not matter so much, the restrictions are just slightly different), namely so as to make the zero energy densities for the vacua.

By providing such restrictions among coupling constants it has the chance to serve as a candidate for solving some fine tuning problems[75, 76]. We could also say that this “Multiple Point Principle” means, that the universe-vacuum is just at some multiple point, where several phases can coexist, much like one at the triple point for water has coexistence of ice, fluid water, and vapor for a common set of intensive variables, pressure and temperature. There may be *no real good derivation* or argument for our “multiple point principle” in spite of the fact, that we have published some attempts to derive this principle [11, 37, 38, 26, 27, 28, 29, 30, 31], but all such arguments would have to involve some influence of the future on the past, at least on the coupling constants, and that would make all such derivations of MPP suspicious. The reader should rather take some previous works - even *prediction(s)* - as well as the results of the present work as *empirical* evidence for this *new law of nature*, the “multiple point principle”(see also [17]).

Our main picture, the consistency of which we suggest should lead to the belief in it, consists of the following three ingredients or assumptions:

- 1. There is very strongly bound bound state of 6 top + 6 anti top quarks (very strongly here means binding energy not small compared to mass-energy of the constituents.),
- 2. our “Multiple Point Principle” saying that there are some *different* ground states (=vacua) of the quantum field theory, all with almost zero energy density(=cosmological constant), say in the Standard Model 3. Here the “almost zero” means that the energy density of these - actually 3 relevant - different vacua are of the order of the energy density as determined astronomically to be of the order of 3/4 of the total energy density in the universe. (Really we formulate and use slightly different versions - and especially different degrees of accuracy - of “Multiple Point Principle” and for instance take it to mean that there are several vacua with the *same* energy density - and not as suggested for the present article that the energy densities are all small just, see the footnote above - and then we even had an argument for how big the energy density of these vacua should be by using an almost supersymmetric vacua as one of the vacua in the flock of degenerate vacua.[23, 24, 25].)
- 3. Pure Standard Model is for our purpose all there is, i.e. no new physics should be strong enough to disturb severely our calculations. So if our paper is sufficiently convincing in its consistency of the mass of the bound state, it should put some limit on how much new physics could be allowed. Only above $10^{18} GeV$ strong new physics can be allowed in our picture. Below it should at least not severely influence the running by renorm group of the Higgs self-coupling λ

The development of the present article is a calculate/estimate of the mass of the bound state from the binding of the constituents mainly by Higgs exchange, but correction from exchange of gluons and of W's and Z's must be included.

Our estimate or calculation is based on an approximate picture of the bound state of the 6 top + the 6 anti top quarks as consisting of a core or bag, in which the normal vacuum

Higgs field is suspended, surrounded by a region around, in which the top and anti top, which are effectively massless inside this core or bag, tunnel out some distance into the surrounding “normal vacuum”, with the usual Higgs expectation value $\langle \phi_H \rangle = 246 \text{ GeV}$.

However, such a picture is far too crude and needs a series of improvements to become more accurate:

- **Tunnelling** We must take into account, that the top and anti top quarks in the “bag”, the region, where the Higgs field is zero, virtually tunnel out of this bag at the surface, and thus in reality are spread over a region, which includes a *rim* around the bag. The extend of this rim is of the order of magnitude of the inverse top quark mass $1/m_t$. In a somewhat ad hoc way we tune in the precise width of this rim. As a first pedagogical exercise we tune in so as to ensure, that in the limit of zero radius of the bag the mass of the bound state resulting goes to the collected mass of the constituents. In the section 4 we improve on this procedure.
- **Gluons** Although one may expect that somehow this bag-model calculation may take into account the binding due to *Higgs exchange*, of course the effect of gluon exchange cannot possibly have been included into that calculation. So a correction to include the effect of the exchange of gluons between the constituent quark and anti quarks has to be performed extra.
- **Eaten** Similarly the exchange of other components of the Higgs field than the “radial” components parallel to the vacuum expectation value must also be taken into account as a correction. Indeed these other components appear in the Standard Model essentially as the longitudinal components of the weak gauge bosons W’s and Z. They have so to speak been “eaten” by these gauge particles. So in reality it is the exchange of the weak gauge particles, for which it is needed to correct.

The main point of the present article is, that this estimate of the mass of the bound state of the 6 top + 6 anti tops from bag-model using the parameters of Higgs interactions as known phenomenologically is in a very similar range as the earlier estimates of this bound state mass based very strongly on the assumption of “Multiple Point Principle” that there shall be several with the present one degenerate vacua. This agreement namely means a test of the “Multiple Point Principle”. When the mass, we shall obtain in the present article, namely fit the needs for the validity of the Multiple Point Principle, then we are in reality confirming this principle.

Although thus it is the main point to suggest, that there is some numerical evidence for the Multiple Point Principle, our result of course has the consequence that we predict there to be a bound state with the mass range resulting. Within the uncertainties in our estimates this mass range would have fitted well with the by now essentially dead F(750). So we predict that one shall find a new particle in LHC with the decay branching roughly as described in our article [6]. Our expectations for the production rate are uncertain, but would have fitted well, if one could already have seen the particle. Although it is also well possible that one should not yet have seen this bound state, it is so that at much improved LHC data it should show up in the future.

In the following section 2 we shall review our model of there existing an exceedingly strongly bound system of 6 top + 6 anti top quarks, and of our “multiple point principle” fine tuning the coupling constants, so that for instance a condensate of the bound state can fill the vacuum and cause a “new vacuum” (“condensate” vacuum) with the energy density just finetuned to be again remarkably small, of the same order as say the astronomical observation of the energy

density(= cosmological constant) of the vacuum, in which we live. (This astronomically observed cosmological constant is quite negligible compared to the energy densities of any significance for high energy physics parameters such as the bound state mass or the Higgs mass). A subsection 2.1 of this section 2 is assigned to our “new law of nature”, the “Multiple point principle”.

In the following section 3 we set up the basic bag-like model, and in the subsection 3.2 we adjust a parameter a so as to obtain at least for the case of a very small bag the mass of the bound state going to the sum of the masses of the constituents, as must be the case. (Really it should be even smaller to be discussed later in 4). In the subsection 3.3 we evaluate the energy density required to put the Higgs field to zero rather than to the usual 246 GeV.

In section 4 we improve on the adjustment which we made in subsection 3.2 by recognizing that the bag first develop when the coupling has become quite strong. In section 5 we then seek to correct the first crude result for the mass of the bound state, first of all for the exchange of weak gauge bosons - especially the zero helicity components called “eaten Higgses” - and the effect of gluon exchange.

In section 6 we seek to convince ourselves and the reader, that the way we corrected for the “eaten ” Higgs exchange is not completely crazy physically. In section 7 we review and comment our result.

2 Bound State Picture and “Multiple Point Principle”

The crucial suggestion behind our bound state model of 6 tops + 6 anti tops is, that, since Higgs exchange like any other even order tensor particle exchange delivers attraction between top and top, or top and anti top, or anti top and anti top as well, we get stronger and stronger binding between the top and anti top quarks the more of them we imagine brought together. It is because the top and anti top are the strongest binding quarks, that this type of binding becomes most relevant for the top and anti top. Now, however, the quarks are fermions and thus you cannot just unlimited clump arbitrarily many, e.g. top quarks, together. Since the top quark has a color degree of freedom taking three values, say: red, blue, and yellow, and a spin degree of freedom, that can be up and down, one can bring up to $3 \times 2 = 6$ top quarks into the same orbital state, but because of fermi-statistics no more. So there can in a single orbital (meaning here a basis wave function for the positional(\sim momentum) degrees of freedom) state be just up to 6 top + 6 anti top. Thereby a closed shell is so to speak formed (in the nuclear physics sense). In the zero Higgs mass approximation, which will be effectively valid, when the size of the bound state - the radius - multiplied by the effective Higgs mass is small, the attraction between the top-quarks or between tops and anti tops is quite analogous to that between an atomic nucleus and an electron. So we can for first orientation use the terminology from the quantum mechanical description of atomic physics. Approximating the bound state, that we suggest to be possible to form from 6 top + 6 anti top by thinking of each top or anti top going around a collected object formed from the other 11 quarks, we can talk about different “orbits” in the atomic terminology of a main quantum number n taking positive integer values and further l (the orbital angular momentum magnitude being $\sqrt{l(l+1)}$) and m (the angular momentum around the quantization axis). As in atomic physics the particles in the $n=1$ orbit are bound strongest. This is analogously to the helium atom having especially high excitation energies(\sim being especially strongly bound relatively to neighbors in the Mandelejev system); we, however, have because of the color factor 3 and both quark and anti quark an especially stable(strongly bound) system being a bound state of 6 top and 6 anti top quarks.

Whether the binding of such a system of 6 top + 6 anti top now is sufficiently strong to even bind to form a resonance, let alone with the rather small mass compared to the collective

mass of 6 top + 6 anti top, $12 m_t = 12 \cdot 173 \text{ GeV} = 2076 \text{ GeV}$, is controversial[102, 103, 104]. However, we think ourselves[72, 73], that making use of a long series of corrections, especially also exchange of the other three components of the Higgs than the as simple particle observed component, we can stretch the uncertainties in the calculation so far as to allow a (very) light bound state to be possibly formed[72]. These other components of the Higgs are really present in the Standard model as W's and Z longitudinal components. We call them "eaten Higgses", but really of course it just means to include (longitudinal components of) weak gauge particle Z and W exchanges.(We believe these longitudinal componets to be actually more important than the transverse components in the binding, but of course for high accuracy even the transverse components should be included).

It is important for our hope, that the bound state can indeed bind so strongly, that it get so tightly bound, that the strong Higgs fields inside the bound state even can modify the effective mass of the Higgs significantly there. We[73] estimated that a top-Yukawa coupling of $g_t = 1.02 \pm 14\%$ would be just sufficient to bind an extremely light bound state of the 6 top and 6 anti top, and would match with the experimental top-Yukawa $g_t = 0.935$. But Shuryac et al. [102, 103, 104] find, that due to the high Higgs mass, it cannot bind at all for the experimental value of g_t .

2.1 MPP

The whole speculation about our bound state of 6 top + 6 anti top is a priori rather much taken out of the air by itself. However, we have all the time proposed it only connected with another speculation, the "Multiple Point Principle". This is, you could say, a wild guess about simplifying or unifying the fine tuning problems of the Standard Model. In order to formulate just the cosmological constant problem[107] about, why the cosmological constant(= the vacuum energy density) compared to say Planck scale dimensional expectations is so enormously small, one needs an *assumption* of the form "The energy density of vacuum is extremely small!" Now you could look at the "Multiple Point Principle" as an *extension* - or putting into "plural" - of this anyway needed assumption, *without* really *complicating* it severely: "Several vacua have extremely small energy densities!". We almost just have put the anyway needed assumption into "plural", or changed the "quantor" from "The physical vacuum..." to "Several vacua...".

Now the real supporting point for this principle is, that although it is not unnecesarily complicated, it is the one, which Colin Froggatt and I managed to use to make historically[16] in 1996, long before the Higgs particle were found, a *prediction* of the Higgs mass of 135 GeV (or 130 GeV) $\pm 10 \text{ GeV}$. Now our prediction using the same Multiple Point Principle would be 129.4 GeV [63] but with a much smaller uncertainty, comparable to the experimental uncertainty of a few hundred MeV. So at first it then looks, that while our original *prediction* agreed perfectly within errors, and the Multiple Point Principle were perfectly right, it is today deviating of the order of three standard deviations (some uncertainty comes from the mass of the top quark, which goes strongly into the Higgs mass prediction) from matching experiment. This formal disagreement of the theoretical prediction actually came in spite of, that the better calculations and better top mass moved our prediction *closer* to the experimental value 125 GeV during the time we had *predited* it. It is of course only possible, that in spite of this development the agreement relative to the uncertainty could become worse, because the uncertainties in calculation and top and Higgs masses went down even faster. However, L.V. Laperashvili, C. Das and myself [8] found, that the existence of the bound state of the 6 top + 6 anti top would make a *little theoretical correction to the mass of the Higgs* being predicted from the multiple point principle, so that the agreement might indeed be improved to be perfect, if the mass of

this bound state is appropriate. According to our estimates a mass about 850 GeV or 710 GeV is what is very crudely called for.

It should be stressed, that this successful Higgs-mass *prediction* as well as Colin D. Froggatts and mine[73, 72] controversial argument, that the top-Yukawa-coupling g_t in order to allow for a condensate of bound states of 6 top and 6 antitop with energy density close to zero, must be close to the value $1.02 \pm 14\%$ supports the “Multiple Point Principle” as being a principle uphold by nature. The value $g_t = 1.02 \pm 14\%$ namely matches with the experimentally determined Higgs Yukawa coupling $g_t = 0.935$. Really we just estimated, what the top-Yukawa coupling should be in order, that the bound state assumed to exist of 6 top + 6 anti top should have exceptionally low mass. But this should be approximately needed to have the condensating particle have mass close to zero in order for there being two degenerate vacua as required by MPP.

This means that even, if the theoretical arguments for the MPP are not totally convincing, there is nevertheless some *empirical evidence* pointing in favor of this MPP. And the present article is meant to provide one more such indirect phenomenological support for MPP.

3 Bag-like Model

3.1 First Charicature

The basic idea of our calculation/estimation of the mass of the hoped for bound state of 6 top and 6 anti top is, that this bound state is in the very crudest approximation a sphere of radius R , inside which the Higgs field ϕ_H is arranged to be fluctuating quantum mechanically around zero, rather than around the usual 246 GeV. Inside this ball or bag of suspended Higgs field the quarks are in principle massless and our picture of the bound state then consists in this inside ball region being filled by the 6 top + 6 anti top quarks, running of course with their by Heisenberg uncertainty principle required momentum in average. Then the procedure is to consider the energy of an ansatz with a given radius R of the bag a function of this radius, and then minimize this energy w.r.t. to the radius. The minimal energy is then the first approximation to the mass of the bound state.

For pedagogical reasons let us first put forward this far too crude model:

The energy for the bound state ansatz is in this first approximation given as the sum of two terms:

- **Bag-constant term** In the next to following section 3.3 we estimate the energy density of a piece of space in which the Higgs field has been imposed to fluctuate around $\phi_H = 0$, i.e. the effective potential for the Higgs field at $\phi_H = 0$,

$$V_{eff}(\phi_H = 0) = \frac{v^2 m_H^2}{8} = 0.132 m_t^4 = 1.18 * 10^8 GeV^4 \quad (1)$$

where we have used

$$\text{Normal Higgs vacuum expectation value } v = \langle \phi_H \rangle = 246 GeV \quad (2)$$

$$\text{Higgs mass } m_H = 125 GeV \quad (3)$$

$$\text{Top quark mass } m_t = 173 GeV \quad (4)$$

and have normalized the effective potential to be zero for the value taken on in the usual vacuum. I.e. we normalized to $V_{eff}(v) = 0$.

Then the energy of the whole bag is

$$E_{bag} = \frac{4\pi}{3} R^3 V_{eff}(0) \quad (5)$$

$$= \frac{4\pi * 0.132 m_t^4}{3} R^3 = 0.553 m_t^4 * R^3 \quad (6)$$

$$= 4.95 * 10^8 GeV^4 * R^3 \quad (7)$$

- **Fluctuation Kinetic Energy** If a particle is distributed evenly over the volume of the bag having radius R the average of the square of its distance from the center of the bag will be

$$\langle \vec{r}^2 \rangle = \frac{\int_0^R r^2 * r^2 dr}{\int_0^R r^2 dr} = \frac{3}{5} R^2 \quad (8)$$

and thus using Heisenberg uncertainty principle in the three dimensional form

$$\langle \vec{r}^2 \rangle \langle \vec{p}^2 \rangle \geq \frac{9}{4} \hbar^2 \quad (9)$$

with the inequality taken as an *equality* we derive, that the average of the momentum squared of the particle/constituent confined in the bag must be at least

$$\langle \vec{p}^2 \rangle \geq \frac{9}{4} * \frac{5}{3R^2} = \frac{15}{4} * R^{-2}. \quad (10)$$

For an in the interior of the bag massless particle one has of course $E_{\text{one particle}} = |\vec{p}|$ and thus a crude estimate of the energy on the average is $E_{\text{one particle}} \approx \sqrt{\langle \vec{p}^2 \rangle}$. Thus we get as an estimate for the (kinetic) energy for 12 constituents, which are inside the bag a lower limit, which is actually crudely an estimate in very first approximation:

$$E_{kin} \approx 12 * \sqrt{\langle \vec{p}^2 \rangle} \quad (11)$$

$$\geq 12 * \sqrt{\frac{15}{4} * R^{-2}} \quad (12)$$

$$= \frac{12 * \sqrt{\frac{15}{4}}}{R} \quad (13)$$

$$= \frac{\sqrt{15 * 9 * 4}}{R} \quad (14)$$

$$= \frac{23.24}{R}. \quad (15)$$

The total energy of the bound state model ansatz is then the sum $E_{bag} + E_{kin}$ given by

$$M_{ansatz} = E_{bag} + E_{kin} \quad (16)$$

$$\approx 0.553 m_t^4 * R^3 + \frac{12 * \sqrt{\frac{15}{4}}}{R}. \quad (17)$$

Now we have to find that value of R which gives the lowest M_{ansatz} and that minimum value of M_{ansatz} should then approximately be the bound state mass. In order to find this minimum we therefore differentiate the expression (17) w.r.t. R :

$$\frac{dM_{ansatz}(R)}{dR} \approx 3 * 0.553 m_t^4 * R^2 - \frac{12 * \sqrt{\frac{15}{4}}}{R^2}. \quad (18)$$

Putting this derivative to zero leads to

$$R^4 = \frac{12 * \sqrt{\frac{15}{4}}}{3 * 0.553 m_t^4} \quad (19)$$

$$= \frac{12 * 1.673}{m_t^4} \quad (20)$$

$$= 12 * 1.303 * 10^{-9} GeV^{-4} \quad (21)$$

$$= 1.564 * 10^{-8} GeV^{-4}. \quad (22)$$

This leads to the radius giving the lowest mass ansatz being

$$R = \left(\frac{12 * 1.673}{m_t^4} \right)^{1/4} \quad (23)$$

$$= 12^{1/4} * 1.137 / m_t \quad (24)$$

$$= 12^{1/4} 6.574 * 10^{-3} GeV^{-1} \quad (25)$$

The procedure to find the mass is to insert this value of R into the expression (17), and that leads to

$$M_{ansatz}|_{min} = 0.553 m_t^4 * (12^{1/4} * 1.137 / m_t)^3 + \frac{12 * \sqrt{\frac{15}{4}}}{12^{1/4} * 1.137 / m_t} \quad (26)$$

$$= m_t * 12^{3/4} * (0.553 * 1.137^3 + \sqrt{\frac{15}{4}} / 1.137) \quad (27)$$

$$= m_t * 12^{3/4} (0.813 + 1.703) \quad (28)$$

$$= m_t * 12^{3/4} * 2.516 \quad (29)$$

$$= 12^{3/4} * 435 GeV \quad (30)$$

$$= 2806 GeV \quad (31)$$

So the mass of the bound state is estimated in this first “calculation” as 2806 GeV. But that is *crazy*, because a bound state should have at most the mass of the collection of constituents

$$\text{Collection of constituent mass} = 12 m_t = 2076 GeV. \quad (32)$$

One mistake we have made, is to assume that the quarks cannot come out of the region (=the bag) in which they are massless, even by tunnelling. But that is of course not true; rather one would expect the quarks to tunnel outside the bag over a length of the order of a top-quark compton wave length $1/m_t$. They would effectively increase the radius to be used for estimating the kinetic energy term E_{kin} by an extra amount of this order of magnitude. Really it means that we must imagine a *rim* around the bag of thickness of this order $\sim 1/m_t$.

But of course the also the Higgs field does *not* jump at the bag-surface as we used at first.

3.2 Adjusting the Tunnelling Around the Bag

The procedure proposed in the present article is to seek to make, what we could call a semi-empirical formula for the mass of the bound state, by adjusting an ad hoc coefficient a to make the result at least so sensible as to give a sensible result in the limit of a small bag - i.e. small R - which corresponds to no effect of the Higgs field and thus the Higgs exchange except the

usual background giving the masses to the quarks. We shall below in section 4 modify this point of view, and indeed rather seek to estimate, that when the bag at all begins to show up in the structure of the bound state, there is already an appreciable binding, and the mass should already be rather $3/4 * \text{“the mass of the constituents”}$. However, to obtain first a relatively simple orientation of the order of magnitudes for the mass obtainable, we shall now calculate with the simple assumption, that the binding just begins at zero radius R , although this is not true. It is at least better than the crazy result of getting a bigger mass than the collected constituent mass.

The more sensible result in this small R limit, which we expect - if it were not for that there could for low binding be no bag at all, but rather only the rim - is that the ground state mass must equal the sum of the masses of the constituents (32). We choose to use as the adjustable parameter the thickness of the rim by putting the correction of the R to be used for the E_{kin} via the Heisenberg uncertainty to be

$$R \rightarrow R + \frac{a}{m_t}. \quad (33)$$

That is to say we shall take instead of (15) the kinetic energy to be

$$E_{kin} \approx 12 * \sqrt{\langle \vec{p}^2 \rangle_{mod}} \quad (34)$$

$$\geq 12 * \sqrt{\frac{15}{4} * (R + \frac{a}{m_t})^{-2}} \quad (35)$$

$$= \frac{12 * \sqrt{\frac{15}{4}}}{R + \frac{a}{m_t}} \quad (36)$$

$$= \frac{\sqrt{15 * 9 * 4}}{R + \frac{a}{m_t}} \quad (37)$$

$$= \frac{23.24}{R + \frac{a}{m_t}}, \quad (38)$$

and then we must adjust a so as to make the whole mass in the limit $R \rightarrow 0$ - but that means effectively in this limit the E_{kin} term alone (because then $E_{bag} \rightarrow 0$) - to be the collected constituent mass $12m_t = 2076 GeV$. Thus we see, that we must choose $a = \sqrt{\frac{15}{4}}$.

With this adjustment we obtain for the ansatz energy of the bag energy plus the kinetic energy

$$M_{\text{ansatz } a = \sqrt{15/4}} = \frac{4\pi}{3} R^3 V_{eff}(0) + 12 * \sqrt{\langle \vec{p}^2 \rangle_{mod}} \quad (39)$$

$$= 0.553m_t^4 * R^3 + 12 * \sqrt{\frac{15}{4} * (R + \frac{\sqrt{15/4}}{m_t})^{-2}} \quad (40)$$

$$= 0.553m_t^4 * R^3 + \frac{12 * \sqrt{\frac{15}{4}}}{R + \frac{\sqrt{15/4}}{m_t}} \quad (41)$$

$$= 0.553m_t^4 * R^3 + \frac{23.24}{R + \frac{\sqrt{15/4}}{m_t}}. \quad (42)$$

Now we must of course as before differentiate this expression (42) w.r.t. R :

$$\frac{dM_{\text{ansatz } a = \sqrt{15/4}}(R)}{dR} = 3 * 0.553m_t^4 * R^2 - \frac{23.24}{(R + \frac{\sqrt{15/4}}{m_t})^2}. \quad (43)$$

Putting the derivative to zero leads to the equation for the R giving the minimal value for the ansatz energy:

$$R^2 \left(R + \frac{\sqrt{15/4}}{m_t} \right)^2 = \frac{23.24}{3 * 0.553m_t^4} = 14.00/m_t^4 \quad (44)$$

Writting explicitley the 12 used as the number of constituents this relation takes the form

$$R^2 \left(R + \frac{\sqrt{15/4}}{m_t} \right)^2 = \frac{12 * \sqrt{15/4}}{3 * 0.553m_t^4} = 12 * 1.167/m_t^4. \quad (45)$$

Taking the square root of both sides of this equation (45) leads to

$$R \left(R + \frac{\sqrt{15/4}}{m_t} \right) = \sqrt{\frac{12 * \sqrt{15/4}}{3 * 0.553m_t^4}} \quad (46)$$

$$= \sqrt{12} * 1.080/m_t^2. \quad (47)$$

This equation is a second order equation in R

$$R^2 + \frac{\sqrt{15/4}}{m_t} * R - \sqrt{12} * 1.080/m_t^2 = 0 \quad (48)$$

and is solved by

$$R = -\frac{\sqrt{15/4}}{2m_t} \pm \sqrt{\frac{15}{16m_t^2} + \sqrt{12} * 1.080/m_t^2} \quad (49)$$

The radius including the tunnelling rim around the genuine bag $R + \frac{\sqrt{15/4}}{m_t}$ thus is

$$R + \frac{\sqrt{15/4}}{m_t} = \frac{\sqrt{15/4}}{2m_t} \pm \sqrt{\frac{15}{16m_t^2} + \sqrt{12} * 1.080/m_t^2}, \quad (50)$$

and numerically we get in units of top-quark masses m_t for the physical solution having the plus sign in front of the square root

$$R + \frac{\sqrt{15/4}}{m_t} = 0.968/m_t + \sqrt{0.9375 + \sqrt{12} * 1.080/m_t} \quad (51)$$

$$= 0.968/m_t + \sqrt{4.678}/m_t \quad (52)$$

$$= 0.968m_t^{-1} + 2.163m_t^{-1} \quad (53)$$

$$= 3.131m_t^{-1} \quad (54)$$

$$\text{while } R \text{ itself is:} \quad (55)$$

$$R = -0.968m_t^{-1} + 2.163m_t^{-1} \quad (56)$$

$$= 1.195m_t^{-1}. \quad (57)$$

Using these results for the radius with and without the rim inclusion we obtain the mass of the ansatz bound state

$$M_{\text{ansatz } a = \sqrt{15/4} |_{\min}} = 0.553m_t^4 * (1.195m_t^{-1})^3 + \frac{12 * \sqrt{\frac{15}{4}}}{3.131m_t^{-1}} \quad (58)$$

$$= 0.553 * 1.195^3 m_t + \frac{12 * \sqrt{\frac{15}{4}}}{3.131} m_t \quad (59)$$

$$= 0.944m_t + 12 * 0.618m_t \quad (60)$$

$$= 0.944m_t + 7.422m_t \quad (61)$$

$$= 8.366m_t \quad (62)$$

$$= 1447\text{GeV} \quad (63)$$

This means that our ad hoc modification of the bag with rim of tunnelling precense of the quarks leads to the mass 1447 GeV for the bound state having used the assumption also of effective equality in the Heisenberg uncertainty relation. This is already an impressive binding, but we have got this value only including the effect of binding from the exchange of what we call the “radial” Higgs, namely the experimentally observed Higgs component. But we have got this mass 1447 GeV not using the gluon exchange force nor what we call “eaten Higgses” meaning essentially, that we did not include the effect of weak gauge boson exchanges.

3.3 Bag Energy Density

For the very low field region, which is relevant for estimating the “bag constant” $V_{eff}(0)$, i.e. the energy density price for removing the Higgs field ϕ_H , the renormalization group running is only of minor significance, and calling the expectation value of the Higgs field in the present vacuum for v we may write the effective potential as

$$V_{eff}(\phi_H) = K(\phi_H^2 - v^2)^2, \quad (64)$$

where then mass square of the Higgs particle equals the second derivative of the potential at the vacuum point $\phi_H = v$:

$$m_H^2 = \frac{d^2}{d\phi_H^2} V_{eff}(\phi_H)|_{\phi_H=v} = K * 2(\phi_H + v)^2|_{\phi_H=v} = 8Kv^2, \quad (65)$$

and thus $K = m_H^2/(8v^2)$, so that the energy density for zero field is

$$V_{eff}(\phi_H = 0) = Kv^4 = m_H^2 v^2 / 8 = (125\text{GeV} * 246\text{GeV})^2 / 8 = (1.014m_t)^4 / 8 = 0.132m_t^4 = 1.1820 * 10^8 \text{GeV}^4 \quad (66)$$

4 Changed Point of View

In the above calculation we effectively assumed, that the weak coupling limit were equal to the limit of the bag radius R going to zero But this is not true because our bag has zero Higgs field in it, while there will in a realistic situation with a weak coupling be no appreciable or dominant region, in which the Higgs field is zero. It will in fact only be close to zero or formally even negative in an extremely small region about each constituent due to the formal divergence of the $1/r$ like Yukawa potential. If one considers that there are several constituents fluctuating

around the potential coming from the sum of these different constituents in a slightly smeared approximation will have no zeros in the weak coupling case at all. So taking a bag with zero field as the major approximation is definitely rather bad for weak coupling. We can thus only hope that this bag-model ansatz can be good for sufficiently strong coupling. But if so, then the adjustment of the “ad hoc” parameter a used above from the requirement, that in the weak coupling case the energy of the bound state should equal the sum of the constituent masses becomes meaningless. If our ansatz does not apply for weak coupling, then we should not use the weak coupling limit to adjust our parameters.

Let us therefore at least in words rather think of doing the following:

We seek to construct an ansatz this way:

- For weak coupling and a long way up in coupling we construct an “ansatz potential” being as well we can describe it at large r (= the distance to the center) the Coulomb or better the Yukawa potential from all the constituents except for the one considered, and then at smaller distances r it continuously gets reduced in strength corresponding to that only a part of the constituents are at shorter distance than r and thus contribute to the Yukawa potential at that distance. Thereby we obtain for the averaged field felt by a considered constituent from the other ones a potential that chopped down or flattened off as r becomes smaller. Because of this flattening off or chop down in magnitude the Higgs field describing approximate potential from the 11 constituents never reaches down to zero.
- Only as we let the coupling be stronger and stronger the in this way constructed potential and corresponding Higgs field will have this Higgs field touch zero, first in the center $r = 0$ of course. But now formally we would like for even stronger coupling to let the Higgs field very near the center - i.e. for r rather small - go negative. But negative Higgs field does not really help attracting a top quark. The point is that the attraction with a Higgs potential really comes about, because the mass of say a top quark is proportional to the numerical value of the Higgs field. But then because of this “numerical” value making the Higgs field negative comes to function rather as a positive potential anyway. Therefore we are stuck by seeking to go further along by making a potential ansatz once we pass the strength at which the Higgs field in the center $r = 0$ becomes zero.
- Now we would get a better binding by letting the Higgs field never go to negative values, but rather stay at zero even though the coupling goes further. That means that from some finite coupling strength at which the first ansatz Higgs field reaches zero further increase of the coupling suggests to have a then larger and larger region with just zero Higgs field, the bag.

This means then that a bag model like ansatz becomes appropriate only after we have reached a so strong coupling situation that the effective field describing the effect of the 11 other constituents on one of them becomes so strong as to have the Higgs field reach zero in the middle.

For the fitting of the parameter a giving width of the rim in units of top-compton wave lengths the just given consideration means that we should not fit the zero-bag radius limit $R \rightarrow 0$ giving the bound state mass equal to the sum of the constituent masses, but rather the mass of the ansatz achieved just when the effective Higgs field describing the potential for one of the constituent just begin to touch zero at $r = 0$. That is to say we must first estimate the bound state mass in the situation, where this effective Higgs field has not yet reached to zero, but just come there to touch zero.

Very crudely we can now say, that the Higgs field in radial direction runs from zero to the usual vacuum expectation value in the transition situation. Very crudely we might then say that the top mass in this range runs from zero to the top mass in the usual vacuum, which is what we call m_t . Then one could say that on the average the mass in this range is $m_t/2$. Now though the top quark present in this region having such a crude half mass also would have a kinetic energy larger than, if it were at rest. From virial theorem one usually take it that in a $1/r$ like potential has the kinetic energy being (minus) the half the potential one. The latter is the difference of the mass that were $m_t/2$ (on average) and the usual mass m_t . So we estimate in the transition case - wherein the effective Higgs field just reach to zero at $r = 0$ - that the bound state mass is 12 times $\frac{3}{4}m_t$. This comes about, because the binding is estimated to $\frac{1}{2} * \frac{1}{2}m_t$ for the transition situation.

We should then in the limit of $R \rightarrow 0$ rather than the mass $12 m_t$ require the mass $12 * \frac{3}{4}m_t$. This corresponds to that the rim is somewhat broader than we assumed above.

This means that now we must in the equation (42) above replace $m_t \rightarrow \frac{3}{4}m_t$ where it goes into giving the thickness of the rim so as to guarantee that in the small R limit we have the suggested mass $\frac{3}{4} * 12m_t$ for the bound state. The value on the other hand of the bag constant is not changed of course.

$$M_{\text{ansatz } a = \sqrt{15/4}, \text{ improved}} = \frac{4\pi}{3}R^3V_{eff}(0) + 12 * \sqrt{\langle \vec{p}^2 \rangle_{mod}} \quad (67)$$

$$= 0.553m_t^4 * R^3 + 12 * \sqrt{\frac{15}{4} * (R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t})^{-2}} \quad (68)$$

$$= 0.553m_t^4 * R^3 + \frac{12 * \sqrt{\frac{15}{4}}}{R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t}} \quad (69)$$

$$= 0.553m_t^4 * R^3 + \frac{23.24}{R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t}}. \quad (70)$$

Differentiating so as to seek the minimum for this by the $\frac{3}{4}$ factor in the top-mass to give the rim thickness gives

$$\frac{dM_{\text{ansatz } a = \sqrt{15/4} \text{ improved}}(R)}{dR} = 3 * 0.553m_t^4 * R^2 - \frac{23.24}{(R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t})^2}. \quad (71)$$

and thus the equation to determine this minimum becomes

$$R^2 \left(R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t} \right)^2 = \frac{12 * \sqrt{15/4}}{3 * 0.553m_t^4} = 12 * 1.167/m_t^4. \quad (72)$$

The roots of the square root of this equations then lead to

$$R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t} = 0.968/(\frac{3}{4}m_t) + \sqrt{0.9375/(\frac{3}{4})^2 + \sqrt{12} * 1.080/m_t} \quad (73)$$

$$= 1.291/m_t + \sqrt{5.408}/m_t \quad (74)$$

$$= 1.291m_t^{-1} + 2.325m_t^{-1} \quad (75)$$

$$= 3.616m_t^{-1} \quad (76)$$

$$\text{while } R \text{ itself is:} \quad (77)$$

$$R = -1.291m_t^{-1} + 2.325m_t^{-1} \quad (78)$$

$$= 1.034m_t^{-1}. \quad (79)$$

Inserting these radii into the mass ansatz we obtain now with rim adjustment made to give $\frac{3}{4}$ of 12 m_t for the bound state mass in the $R \rightarrow 0$ limit:

$$M_{\text{ansatz } a = \sqrt{15/4} \text{ improved}}|_{\min} = 0.553m_t^4 * (1.034m_t^{-1})^3 + \frac{12 * \sqrt{\frac{15}{4}}}{3.616m_t^{-1}} \quad (80)$$

$$= 0.553 * 1.034^3 m_t + \frac{12 * \sqrt{\frac{15}{4}}}{3.616} m_t \quad (81)$$

$$= 0.612m_t + 12 * 0.535m_t \quad (82)$$

$$= 0.612m_t + 6.42m_t \quad (83)$$

$$= 7.03m_t \quad (84)$$

$$= 1216\text{GeV} \quad (85)$$

5 Corrections

In addition to the simple Higgs exchange, on which we have concentrated above, there is in the type of bound state, we consider also gluon exchange and exchange of what we called “eaten Higgses”, but which are really just exchange of weak gauge bosons with longitudinal polarization. In our work [73] we estimate the effect of such corrections on the critical coupling value $g_{t \text{ crit}}$ at which a phase transition between the usual vacuum and a vacuum with a condensate or at least some filling with the bound states to be a factor $4^{1/4}$. indeed we estimate that the “eaten Higgs” extra Higgses function crudely in the approximate situation of the bound state being very light as if the g_t had been increased by the fourth root of the ratio of the new number to the old number of Higgs components, i.e. the fourth root of 4. Expressed for the square of the Yukawa coupling g_t^2 this effect of the “eaten Higgses” becomes thus a factor 2. Similarly one finds in our article[73] that the inclusion of the gluon interaction in addition to the Higgs exchanges has an effect of the replacement:

$$4g_t^2 \rightarrow 4g_t^2 + 1.83. \quad (86)$$

Since $g_t^2 = 0.935^2 = 0.874$ we have $4g_t^2 = 3.50$, and the addition of the gluon correction corresponds to an increase of g_t^2 by a factor $1 + \frac{1.83}{4*0.874} = 1 + 0.523 = 1.523$.

Now we can argue, that the bag constant could be considered to be inversely proportional to the 4th power of the top-Yukawa-coupling g_t in a notation, in which mass of the top m_t is put to be independent of this Yukawa coupling:

$$V_{eff}(0) = 0.0323v^4 = \frac{0.0323 * m_t^4 * 4}{g_t^4} = 0.1292 * \frac{m_t^4}{g_t^4} \quad (87)$$

The collective effect crudely of both “eaten Higgses” and the gluon exchanges corresponds to a replacement of the square of the Yukawa coupling by a factor $2 * 1.523 = 3.05$. This then corresponds to decreasing the bag constant by a factor $3.05^2 = 9.28$.

This bag constant goes into the above as the inverse square root in the constant term in the quadratic equation of the type of the square root of equation (72) such as say (47) or (48).

This constant term is seen from (74) to then change as

$$\sqrt{12} * 1.080m_t^{-2} \rightarrow \sqrt{12} * 1.080m_t^{-2} * 3.05. \quad (88)$$

Thus we get for the radius including and not including the rim:

$$R + \frac{\sqrt{15/4}}{\frac{3}{4}m_t} = 0.968/(\frac{3}{4}m_t) + \sqrt{0.9375/(\frac{3}{4})^2 + \sqrt{12} * 1.080 * 3.05/m_t} \quad (89)$$

$$= 1.291/m_t + \sqrt{13.06}/m_t \quad (90)$$

$$= 1.291m_t^{-1} + 3.614m_t^{-1} \quad (91)$$

$$= 4.905m_t^{-1} \quad (92)$$

$$\text{while } R \text{ itself is:} \quad (93)$$

$$R = -1.291m_t^{-1} + 3.614m_t^{-1} \quad (94)$$

$$= 2.32m_t^{-1}. \quad (95)$$

Now in calculating the mass of the bound state after the correction of g_t^2 by the factor 3.05 we shall remember that the bag constant going into the term $\frac{3\pi}{4}R^3V_{eff}(0)|_{after\ correction}$ in the bound state mass has to be reduced compared to the genuine $V_{eff}(0)$ by a factor 3.05^2 .

So this bag-model term becomes

$$\frac{4\pi}{3}V_{eff}(0)_{reduced}R^3 = \frac{4\pi}{3}*(1.014m_t)^4/8/3.05^2(2.32m_t^{-1})^3 = 0.553/3.05^2*2.32^3m_t = 0.0594*2.32^2m_t = 0.320m_t. \quad (96)$$

This term is rather small. Here we used:

$$V_{eff}(\phi_H = 0) = Kv^4 = m_H^2v^2/8 = (125GeV*246GeV)^2/8 = (1.014m_t)^4/8 = 0.132m_t^4 = 1.1820*10^8GeV^4 \quad (97)$$

The term from the kinetic energy mainly of the top-quarks and anti-top-quarks is now given as

$$\frac{12 * \sqrt{15/4}}{4.905m_t^{-1}} = 12 * 0.395m_t = 4.74m_t. \quad (98)$$

Together with the small bag-term this gives very crude corrected mass for our bound state of 6 top and 6 anti top

$$M_{corrected\ eaten+gluons} = 0.320m_t + 4.74m_t = 5.06m_t = 875GeV. \quad (99)$$

This would agree wonderfully with the mass of the F(750) if it would resurect! More importantly: It also agrees very well with estimates for the needed mass of the bound state to just make the correction to the vacuum energy density at the high higgs field minimum at $\phi_H \sim 10^{18}GeV$ so that it corrects the present value[8] of the selfcoupling $\lambda(10^{18}GeV)$ from its -0.01 ± 0.002 to just 0:

$$\begin{aligned} m_F &\approx \frac{6g_tm_t}{b} \left(\frac{\sim 2}{\pi^2 * 0.01 \pm 0.002} \right)^{1/4} \\ &\approx 2.31 * 173GeV * 2.1 = 4.9 * 173GeV = 850GeV \pm 20\% \\ \text{or without the } \sim 2: m_{F(750)} &= 2.31 * 173GeV * 1.8 = 4.1 * 173GeV = 710GeV \pm 20\% \end{aligned}$$

Further it fits very well with my earlier simple calculation [9], which gave a mass of $4m_t = 692\text{GeV}$ and which were based on assuming the “condensate vacuum” being degenerate with the “present” vacuum.

6 Some Worries, Can we Trust?

Before believing the story that we could correct for the gluon exchange and eaten Higgs exchanges just by replacing the bag constant $V_{eff}(0)$ by a dramatically smaller value - indeed diminished by a factor 3.05^2 - and thus obtain a mass correction

$$1216\text{GeV} = 7m_t \rightarrow 5m_t = 875\text{GeV} \quad (100)$$

we should seek to understand: How can this be understandable physically? Well, one would say, that when the top and anti top particles are present in the bag, then they interact with their neighbors by means of gluon exchange and eaten Higgs exchange, and thus the energy density of a bag filled with top and anti tops has indeed a reduced energy density. It would be nice to check that this reduction can be so large as we used above.

If we consider the situation inside the “bag” where the Higgs mass is effectively zero, the mass of the weak gauge bosons must also be zero. So as long as we consider this “inside” the top and anti top quarks are attracted by exchange of these gauge bosons as if they were massless. Also here the mass plays no role and there is actually here no essential difference between the left b and the left top. So in the “inside” region there should ideally be about equally many left handed top and left handed bottom. But right ones are only top-right, because the right bottom is in our approximation totally decoupled. Very crudely we might say that a replacement of half the amount of the left top-quarks by bottoms means, that the number of quarks present per volume unit up to a given energy height gets increased by a factor $3/2$. If we thus want to have 12 quarks, this would in a calculation, in which we did not have this effect of the W and Z included, mean that we only should require place for $12:(3/2) = 8$ quarks instead if using our calculations above. In fact we could claim for each 12 top or anti top, 3 could hide as left bottom quarks. Crudely we could for a given bag size and given number of quarks decrease the needed kinetic energy per quark by a factor corresponding to that the quark-particle density could be decreased by the factor $3/2$.

We might therefore instead of including the effect as we did by changing the effective bag-constant take instead an effective number 8 for the number of quarks in the bound state. Now above we found without the corrections

$$M_{without\ correction} = 7.03m_t = 1216\text{GeV}. \quad (101)$$

In the crude thinking that the term with the kinetic energy dominated and that this term depends crudely proportionally to $3/4$ ’th power of the number of particles 12. So if we reduce this 12 to 8 then the mass of the bound state should go down by the factor $(12/8)^{3/4} = 1.36$, and thus we would in this way get a mass around

$$7.03m_t/1.36 = 1216/1.36\text{GeV} = 5.17m_t = 897\text{GeV}. \quad (102)$$

This bound state mass to be fully corrected should still be corrected for the gluon contribution to the attraction.

Above we saw that the gluon correction were about a factor 1.523 counted in the g_t^2 . Saying e.g. that the binding energy in the Bohr atom goes as the coupling to the fourth power, the binding should be increased by $1.523^2 = 2.32$ due to the gluon exchange.

On a logarithmic scale this correction factor 1.523 is $\frac{\ln(1.523)}{\ln(1.523*2)} = 0.377$, so that so to speak 37.7 % of the correction $1216\text{GeV} = 7m_t \rightarrow 5m_t = 875\text{GeV}$ is due to the gluons, while the remaining 62.3 % is due to the eaten Higgs effect, which we have just replaced by its effect of replacing some of the top or anti topquarks by left handed bottom quark or antibottom quarks. Crudely we would estimate that the correction due to the gluons in the mass for the bound state which we found were a factor $(\frac{875}{1216})^{.377} = 0.88$ and thus we should to correct for also the gluon effect diminish the 897 GeV just obtained doing only a replacement for the eaten Higgs correction by further 15 %. Thus we get the new estimate

$$m_{\text{alternative estimate}} = .88 * 897\text{GeV} = 792\text{GeV} \quad (103)$$

7 Conclusion

We have made a crude estimate of the mass of the bound state of 6 top + 6 anti top quarks, about which we have long speculated, that it is very strongly bound, to be crudely

$$m_{\text{bound state}} = 875\text{GeV} \text{ or } 792\text{GeV} \pm \text{say } 40 \%. \quad (104)$$

Our method were mainly a bag-model estimation, in which the bag meant a region, where the Higgs field were reduced to ~ 0 .

The greatest importance of this estimate is, that it remarkably coincides with *two* earlier calculations *based on the assumption, that two speculated vacua potentially existing in pure Standard Model should have degenerate energy densities(= cosmological constants)*. In fact the present author recently found [9] based on a type of calculation first developped in the work with C.D. Froggatt [39] that the mass of the bound state needed for the degeneracy of the speculated phase with a condensate or at least a higher concentration of these bound states with the present vacuum were

$$m_{\text{from "condensate" vacuum}} = \frac{12m_t}{3} = 4m_t = 692\text{GeV}. \quad (105)$$

The second estimate of the bound state mass agreeing remarkably well with the present calculation were in collaboration with Das and Laperashvili [8] and based on the requirement that there should be a vacuum - which we call "the High field vacuum" - for the Higgs field being of the order of 10^{18}GeV having with high accuracy very small cosmological constant or energy density like the present vacuum. This is equivalent to the requirement that the instability of the present vacuum seemingly resulting in pure Standard Model [63] by means of our speculated bound state just gets corrected to be almost exactly on the border line of stability, and it leads to the mass for the bound state

$$m_{\text{bound state}}(\text{from "high field vacuum"}) \approx 850\text{GeV} \pm 30\% \text{ with } \sim 2 \quad (106)$$

$$m_{\text{bound state}}(\text{from "high field vacuum"}) \approx 710\text{GeV} \pm 30\% \text{ without } \sim 2. \quad (107)$$

The two calculations cited here just deviate by including crudely("with ~ 2 ") or not including("without ~ 2 ") some higher diagrams for the bound state causing a correction in the value for the effective self-coupling λ of the Higgs to be used for getting the Higgs mass as to be observed.

Summarizing we have estimated -although very crudely only - the mass of the bound state in *three a priori quite different ways* as put in this table:

Used	mass	In m_t units	Deviation from average	Gussed Uncertainty
Bag-model	830 GeV	~ 5	3 %	40 %
$\Lambda_{present} = \Lambda_{condensate}$	690 GeV	4	(-)8 %	40 %
$\Lambda_{present} = \Lambda_{high\ field}$	780 GeV	4.5	4 %	30 %
Average	770 GeV	4.3		21 %

The agreement of these mass estimates with each other is too good compared to the guessed uncertainties of our calculations, but the latter was not made carefully. If this agreement is taken seriously, it means that the degeneracies of the vacua as implicated by our principle “Multiple Point Principle” could be claimed to have been tested by direct calculation using the parameters of the pure Standard Model.

That would then mean that we would have derived:

- The validity in the case of the three vacua suggested for the pure Standard Model the *new law of nature MPP* !
- It would be suggested that no new physics should come in to disturb the Standard Model more than to not disturb the energy differences to our accuracy relative to those in the pure Standard Model.
- The bound state - that shall do the job - should really exist!
- The mass of it must be our estimated $770\text{ GeV} \pm 19\%$. (It is really sad for our picture that the enhancement known as $F(750)$ with just the right mass and decaying into two gammas, were washed out so that no statistics remains supporting it! We miss it!)

It should be stressed that in principle - i.e. if we can perform non-perturbative calculations sufficiently accurately - we should be able to simply calculate, if there exist the above posulated vacua. Well, the “high field one” confrontation with MPP requires that one presupposes that the Standard Model to be valid to sufficient accuracy all the way up to about 10^{18} GeV . Even for the “condensate vacuum” one could imagine that new physics might modify our calculations, but LHC has already put severe limits telling, that there is no new physics up to a scale of the order of one TeV and thus our proposed bound state of a mass of the order of 770 GeV is expected to be not very sensitive to at present acceptable new physics. Thus improved calculational methods for non-perturbative effects, especially strongly bound states, should possibly rather independently of possible new physics be able to settle, if our bound state of a mass in the range near the value of 750 GeV really exists or not.

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Astri Kleppe even presented our “Mutiple Point Principle” for the workshop in Bled.

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